

**Solutions to the Review Questions at the End of Chapter 2**

1. (a) E(*ax* + *by*) = *a*E(*x*) + *b*E(*y*).

(b) E(*axy*) = *a*E(*x*)E(*y*).

(c) E(*axy*) = *a*E(*xy*).

2. (a) The pdf (probability density function) describes how likely it is that a random variable following a given distribution (e.g., the normal) will take on a value within a given range. The cdf (cumulative distribution function), on the other hand, is a function giving the probability that the random variable will take on a value lower than some pre-specified value (call this *z*). The cdf gives the area under the pdf from minus infinity up to that point *z* and thus we can think of the cdf as the integral of the pdf whereas the pdf is the derivative of the cdf. To illustrate, a pdf could give the probability that a share price will increase by between 10% and 20% in a given year, while the cdf could give the probability that the increase will be less than 5%.

(b) The pdf has a ‘bell shape’, while the cdf is a sigmoid (S-shape).

3. The central limit theorem (CLT) states that the mean of a sample of data having any distribution converges upon a normal distribution as the sample size tends to infinity. This is an important result in statistics since it shows that even if the raw data are not normally distributed, the distribution of their mean will converge on a normal as the sample increases. Hence the conventional approach to statistics involving hypothesis testing using the normal or *t*-distribution tables can be applied. Without the CLT, it would not be valid to use these tables to find critical values.

4. All three are measures of central tendency – i.e. they capture the average or ‘typical’ behaviour of a series. The (arithmetic) mean is simply calculated as the sum of all values in a series divided by the number of values, whereas the mode measures the most frequently occurring value in a series and the median is the middle value in a series when the elements are arranged in an ascending order. It is not possible to say that one of these three measures is better than the other two – each has its own advantages and disadvantages. The benefit of the mean is that it fully encapsulates the information from all data points in the series and it is the most familiar method to most researchers, but can be unduly affected by extreme values and in such cases, it may not be representative of most of the data. The mode is arguably the easiest to obtain, but is not suitable for continuous, non-integer data (e.g., returns or yields) or for distributions that incorporate two or more peaks (known as bimodal and multi-modal distributions, respectively). The median is often considered to be a useful representation of the ‘typical’ value of a series, but has the drawback that its calculation is based essentially on one observation. Thus if, for example, we had a series containing 10 observations and we were to double the values of the top three data points, the median would be unchanged.

5. Again, it is not possible to state that one approach is always better than the other – it really depends on what the estimated mean will be used for. Geometric returns are harder to calculate but give the fixed return on the asset or portfolio that would have been required to match the actual performance, which is not the case for the arithmetic mean. Thus, if you assumed that the arithmetic mean return had been earned on the asset every year, you would not reach the correct value of the asset or portfolio at the end. But it could be shown that the geometric return is always less than or equal to the arithmetic return, and so the geometric return is a downward-biased predictor of future performance. Hence, if the objective is to summarise historical performance, the geometric mean is more appropriate, but if we want to forecast future returns, the arithmetic mean is the one to use.

6. Not necessarily. The covariance between two random variables, call them *x* and *y*, will scale with *x* multiplied by *y*. So, if we multiplied all of the values in a series *x* by 10, the covariance would also increase by a factor of 10 but the variables would not really be any more closely related than they were before. Thus the numerical value that the covariance takes does not have a straightforward interpretation and 0.99 may or may not indicate a high degree of association depending on the scaling of the data. If the correlation figure was 0.99, by contrast, this would be a clear demonstration that the two series are strongly positively correlated since the correlation measure is constructed so that it must lie between –1 and +1.

7. (a) Continuous data come from series that can take on any value (possibly within a given range) and can be measured to any arbitrary degree of precision such as the weight of a lump of cheese or the average return on a stock, but discrete data can only take certain specific values – for example, the number of houses in a street.

(b) Ordinal data arise where a variable is limited so that its values define a position or ordering only, and thus the precise values that the variable takes have no direct interpretation – for example, the performance ranking of a mutual fund among a set of 20 such funds. Nominal data, by contrast, occur when there is no natural ordering of the values at all, so a figure of 12 is simply different to that of a figure of 6, but could not be considered to be better or worse in any sense. Such data often arise when numerical values are arbitrarily assigned, such as telephone numbers or when codings are assigned to qualitative data (e.g., when describing the exchange that a US

stock is traded on, ‘1’ might be used to denote the NYSE, ‘2’ to denote the NASDAQ and ‘3’ to denote the AMEX).

(c) Time-series data are data that have been collected over a period of time on one or more variables – for example, a series of house prices, observed monthly for ten years in a particular region. Thus there is no cross-sectional element in this case so there is a single entity being examined – so one country, one stock, one firm, etc. Panel data, by contrast, simultaneously have the dimensions of both time series and cross-sections, e.g., the daily prices of a number of blue chip stocks over two years. Here we have both many time points and (days) many entities (firms) in the sample.

(d) The distinction between noisy and clean data is a subtle one. In general, ‘noisy’ refers to data that have a large amount of random variation which is considered an uninteresting feature that might get in the way of uncovering the underlying behaviour. The noise might simply be random variation in a series due to its volatility, or it might occur as a result of recording or measurement errors. To a large extent, almost all series that we encounter in economics and finance are noisy. Clean data refers to series where the amount of noise is at a minimal level and the data are at least free of errors.

(e) The main difference between simple and continuously compounded returns are the following: (i) continuously compounded returns are additive across time while simple returns are not and (ii) simple returns are additive across assets but continuously compounded returns are not (that’s because the log of a sum is not the same as the sum of logs). Therefore, computing cumulative returns (across time) is an easy task when one uses simple returns but a difficult one when one uses continuously compounded returns. When building a portfolio, one has to be careful to use simple returns (i.e., apply the weights to the simple returns) given that continuously compounded returns are not additive across assets.

(f) A real series is one that is net of the effect of inflation. Deflating by an asset’s price by an index of aggregate prices in an economy yields a real price for the asset. The real return on an asset is equal to the nominal return on the asset minus the inflation rate.

(g) These are different philosophical approaches to how the data are used in building models. Classical statistics involve building a theoretical model first and then ‘showing it to the data’ with the parameter estimates being freely determined by the data. Bayesian statistics, on the other hand, involve the data and theory working more closely together. The researcher would start with an assessment of the existing state of knowledge or beliefs, formulated into a set of probabilities. These prior inputs or priors would then be combined with the observed data via a likelihood function. The beliefs and the probabilities would then be updated as a result of the model estimation, resulting in a set of posterior probabilities. Probabilities are thus updated sequentially, as more data become available.

8. There is certainly no shortage of possible examples that could be listed here. It is important to note that in many instances a specific problem could be tackled using time-series or cross-sectional or panel data, although it might be that one approach would be more insightful than the others. The following examples were used in the book.

Problems that could be tackled using time-series data

* How the value of a country’s stock index has varied with that country’s macroeconomic fundamentals.
* How the value of a company’s stock price has varied when it announced the value of its dividend payment.
* The effect on a country’s exchange rate of an increase in its trade deficit.

Problems that could be tackled using cross-sectional data:

* The relationship between company size and the return to investing in its shares.
* The relationship between a country’s GDP level and the probability that the government will default on its sovereign debt.

Potentially, any of the above issues that could be considered in the time-series or cross-sectional frameworks could also be tackled in a panel context.

9. Asset return time-series have a number of stylised features that are common whether they are referring to stocks, bonds, house prices, etc. The key ones are

* There is a lot of data available!
* They are noisy and volatile.
* They are leptokurtic and have fatter tails than a normal distribution with the same mean and variance.
* Most such series are negatively skewed, so that large negative returns are more likely than positive returns of the same magnitude.
* They exhibit volatility clustering, so there are bursts where the series is highly volatile for a protracted period and also quiet periods where there is nothing going on for a while.
* They can often best be characterised as a random walk with drift process.

10. These calculations are probably best done in a spreadsheet. If we did so, we would get the following table

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Year | Bond Value | CPI | Simple Returns (in %, part (a)) | Continuously Compounded Returns  (in %, part (b)) | Bond Price in 2018 Terms | Inflation (in % computed as (CPIt-CPIt-1)/CPIt-1) | Real Return (Simple Return -Inflation) |
| 2011 | 36.9 | 108 |  |  | 42.8 |  |  |
| 2012 | 39.8 | 110.3 | 7.86 | 7.57 | 45.2 | 2.13 | 5.73 |
| 2013 | 42.4 | 113.6 | 6.53 | 6.33 | 46.8 | 2.99 | 3.54 |
| 2014 | 38.1 | 116.1 | -10.14 | -10.69 | 41.2 | 2.20 | -12.34 |
| 2015 | 36.4 | 118.4 | -4.46 | -4.56 | 38.6 | 1.98 | -6.44 |
| 2016 | 39.2 | 120.9 | 7.69 | 7.41 | 40.7 | 2.11 | 5.58 |
| 2017 | 44.6 | 123.2 | 13.78 | 12.91 | 45.4 | 1.90 | 11.87 |
| 2018 | 45.1 | 125.4 | 1.12 | 1.11 | 45.1 | 1.79 | -0.66 |

A few notes

The simple returns are calculated as: *returnt* = 100×((*Pt–Pt*–1)/*Pt*–1)

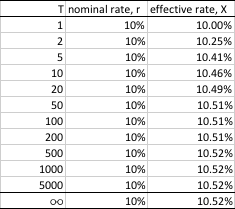
The continuously compounded returns are calculated as: *returnt* = 100×log(*Pt*/*Pt*–1)

The bond prices in 2018 terms are calculated as: *pricet* = *nominal pricet*×*CPI*2018/*CPIt*

The inflation rate is calculated as *inflation ratet* = 100×((*CPIt – CPIt*–1)/ *CPIt*–1)

The real returns are calculated as: *nominal* (simple) *returns* (%) – *inflation rate* (%).

11. Questions 11–14 are mostly straightforward applications of the relevant formulae from the chapter that are best tackled using a spreadsheet.



As the table shows, the effective rate increases as the compounding frequency increases, but at a decreasing rate so that compounding 500 times a year and continuously compounding produce effective interest rates that are identical to two decimal places.

12. (a) The relevant formula to use for this question is

years to two decimal places, where *Z* is the factor that we want the sum of money to increase by (2 for doubling) and *r* is the interest rate expressed as a proportion, 0.05.

(b) All that changes from part (a) is *r*, which is now 0.05. This reduces the time to double the money to 14.21 years.

(c) Now *Z* = 3 and *r* = 0.05 in the same formula, which gives *T* = 22.52 years.

(d) The easiest way to answer this question is to calculate the effective simple interest rate that would be equivalent to a continuously compounded rate of 5% per annum. This is calculated using

which gives a rate of 5.13%. We can then use the same formula as for part (a) but with *Z* = 2 and *r* = 0.0513, which gives 13.86 years to double the money.

13. The easiest way to answer this question is, as for 12(d), to calculate the effective simple interest rate that would be equivalent to a continuously compounded rate of 11% per annum. This is given by

Thus the continuously compounded account pays an equivalent once-per-year rate of 11.57% and so still the 12% account, paid only annually, is still preferable.

14.(a). This question is slightly tricky. One way to solve it is to use a rearrangement of the formula for the sum of *n* terms of a geometric progression. Given that the compounding is only annually, it is easier to work with annual figures rather than monthly, and thus the amount saved is £200 × 12 = £2400 per year. This is a series that we want to sum, which starts with the value at the end of year one: £2400 × (1 + 0.05) and each term will increase by a factor of (1 + 0.05), and thus *a* = 2400(1+0.05), *d* = (1+0.05), *Sn* = £1000000

which can be rearranged as

and thus

So

Placing the numbers into the formula gives

So it would take 62.24 years to make the million pounds, probably out of your reach unless you started saving very early!

(b) If you were able to save more: £500 per month (= £6000 per year), then the same approach would be used as for part (a) but £2400 would be replaced by £6000. This reduces the time taken to get to a million pounds to 44.89 years – roughly the length of time most people save for their pensions.

(c) If we are interested in using the real interest rate (2%) rather than the nominal one (5%), then again everything would be as in part (b) but we would use 1.02 in place of 1.05. In this scenario, it would take 73.28 years to get to the million pounds.